A MATHEMATICAL FUNCTION TO DESCRIBE LACTATION CURVES OF COWS USED IN MEDIUM AND SMALLHOLDER DAIRY SYSTEMS IN KENYA

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ABSTRACT

Inconsistent recording that is characteristic in low to medium input production systems in Sub-Saharan Africa (SSA), pose a great challenge to animal performance evaluation. This study aimed at determining a mathematical function to describe lactation curves of cows in smallholder dairy systems in Kenya to enable accurate predictions of daily and total lactation milk yield from incomplete data. Six lactation functions namely the incomplete gamma function of Wood (WD), exponential function of Wilmink (WIL), Mixed algorithm of Guo and Swalve (GUOS), polynomial regression function of Ali and Schaeffer (ALIS) and mechanistic functions of Dijkstra (DIJ) and Pollott (APOL) were fitted on milk yield records of Jersey, Guernsey and Sahiwal cows. Functions were compared using the adjusted R squared, percentage of squared bias and correlation between the predicted and actual milk yield. The DIJ and WD functions fitted the data better. The DIJ function had high data requirements depicted by convergence failure in some of its analyses. Prediction accuracy varied between breeds and lactations indicating their independence. The WD function was more desirable for modelling lactation curves due to is adaptability to inconsistent recording situations common in smallholder systems.

Key words: Lactation curves; dairy cattle; smallholder production systems

INTRODUCTION

In low to medium input production systems in Sub-Saharan Africa (SSA), animal recording is characterised by inconsistent records (FAO, 1998). These inconsistencies pose a great challenge to animal performance evaluation since they affect the availability and quality of data. Modelling of lactation curves enable accurate predictions of daily and total lactation milk yield from incomplete data (Olori et al., 1999). It also enables inclusion of part lactations in the evaluations. Mathematical functions have been used to describe lactation curves. These functions aim at predicting the yield on each day with minimum error (Macciotta et al., 2005). Studies on accuracy of the functions to describe the lactation curves have shown no agreement on a function offering the best fit across datasets. Therefore, prior to adoption of a function for use in evaluation, several models should be compared for their ability to fit the available data. This study aims at establishing a suitable lactation function to describe the dairy cattle data in order to facilitate inclusion of partial lactations in national dairy cattle evaluation.

MATERIALS AND METHODS

Test day milk yield (TDMY) records of Jersey, Guernsey and Sahiwal were obtained from the national dairy cattle database at the Livestock recording Centre (LRC) in Naivasha, Kenya. Data structure and summary statistics are presented in Table 1. Six lactation functions were fitted to the data using an iterative nonlinear curve fitting procedure applying a Marquardt algorithm computational strategy (PROC NLIN of SAS; SAS, 2004) to determine the function that best described the lactation curve of the dairy cows. The functions included:

a). the incomplete gamma function (WD) described by Wood (1967)

$$Y_t = at^b e^{-ct} \qquad (1)$$

where Y_t is the test day milk yield at DIM t in all the functions, a, b and c in function (1) are parameters representing a scaling factor associated with initial milk yield, the pre-peak and post-peak curvatures, respectively.

b). the exponential function (**WIL**) described by Wilmink (1987)

$$Y_t = a + be^{-kt} + ct \qquad (2)$$

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where a, b and c are parameters associated with the level of production, increase of production pre-peak and the subsequent post peak decrease, respectively. Parameter k assumes a fixed value derived from preliminary analysis and is associated with time of peak lactation (Wilmink, 1987).

c). the Mixed logarithm function (**GUOS**) of Guo and Swalve (1995)

$$Y_t = a + b\sqrt{t} + c\ln(t) \qquad (3)$$

Where a, b and c are the parameters.

d). the polynomial regression function (**ALIS**) of Ali and Schaeffer (1987)

$$Y_t = a + b\gamma + c\gamma^2 + d\omega + \rho\omega^2 \qquad (4)$$

where $\gamma = t/305$, $\omega = \ln(305/t)$, a is a parameter associated with the peak yield, d and ρ are associated with the ascending part of the curve, and b and c are associated with the descending curvature.

e). the mechanistic function (**DIJ**) of Dijkstra *et al.* (1997)

$$Y_t = a\exp\left[\frac{b\left(1-e^{-ct}\right)}{c} - dt\right] \qquad (5)$$

f). the additive form of the mechanistic function (**APOL**) by Pollott (2000).

$$Y_t = a\left[\frac{1}{\left(1+\frac{1-b}{b}e^{-ct}\right)} - \frac{1}{\left(1+\frac{1-d}{d}e^{-gt}\right)}\right]\left(1-e^{-ht}\right) \qquad (6)$$

The ability of functions to fit the data was compared using adjusted coefficient of determination (R^2_{adj}). Lactation functions which had R^2_{adj} values greater than 0.75 were tested for their predictive ability. The accuracy of prediction was evaluated by examining: i) the mean square prediction error (MSPE) ; ii) the correlation between true milk yield and predicted milk yield (r) and iii) the percentage of squared bias (PSB Ali and Schaeffer, 1987).

TABLE I- STRUCTURE AND SUMMARY STATISTICS OF DATA COLLECTED FROM LRC

Dataset[a]	No. of herds	Breed	No. of animals	No. of records	Ave. no. of records/ parity	Mean test day milk yield	Std Dev
DS-1	155	Guernsey	270	5391	898.50	10.06 (0.05)	3.44
		Jersey	701	18499	3083.17	13.53 (0.04)	5.42
		Sahiwal	541	6803	1133.83	3.96 (0.03)	2.39
DS-2	137	Guernsey	167	2700	450.00	10.75 (0.06)	3.31
		Jersey	518	9730	1621.67	14.24 (0.05)	5.17
		Sahiwal	207	2350	391.67	4.25 (0.05)	2.34
DS-3	155	Guernsey	269	4826	804.33	10.41 (0.05)	3.34
		Jersey	699	16463	2743.83	14.00 (0.04)	5.26
		Sahiwal	541	6429	1071.50	4.10 (0.03)	2.36
DS-4	137	Guernsey	167	1890	315.00	9.78 (0.07)	2.84
		Jersey	518	6811	1135.17	13.32 (0.06)	5.12
		Sahiwal	207	1645	274.17	3.50 (0.05)	1.93
DS-5	137	Guernsey	167	1890	315.00	11.15 (0.08)	3.45
		Jersey	518	6811	1135.17	14.62 (0.06)	5.22
		Sahiwal	207	1645	274.17	4.58 (0.06)	2.45

[a]DS-1= a 12 test day dataset with randomly missing test day records; DS-2= a 10 test day dataset without missing test day records; DS-3= a 10 test day dataset with randomly missing test day records; DS-4= 7 test day record dataset (Milk records taken from the 4th month post partum); DS-5= a 7 test day dataset (test day 1,2,3 and 4 taken on a monthly basis post partum and test day 5, 6 and 7 taken bimonthly).

RESULTS AND DISCUSSION

Estimates of R^2_{adj} for preliminary analysis to determine the fit of the functions on the five datasets are presented in Table 2. Only WD, DIJ and APOL functions had R^2_{adj} values large than 0.75 implying the functions offered the best fit to the data. However, cases of difficult convergence were noted in DIJ and APOL functions' runs attributable to data limitations. These indicate their high data requirements. Most of APOL runs failed to converge leading to it being dropped from further consideration. Other functions (WIL, GUOS and ALIS) had R^2_{adj} values less than 0.127. WD and DIJ functions competed closely with minimal differences between $R^2_{adj.}$ Therefore, only WD and DIJ functions were selected for evaluation of predictive ability.

TABLE II- ESTIMATES OF ADJUSTED COEFFICIENT OF DETERMINATION (R^2_{ADJ}) FOR THE MODELS FITTED TO THE DATA

Function	WD	WIL	GUOS	ALIS	DIJ	APOL
R^2_{adj} estimate	0.811	0.126	0.126	0.126	0.811	0.812

The prediction ability of the WD and DIJ functions are presented in Table 3. Mean squared prediction error (MSPE) between models varied slightly with a difference of less than 0.1 in favour of DIJ function. The superiority of DIJ function over WD function was also reported in a study lactation curve of dairy cattle in Mexico (Val-Arreola et al., 2004). Percentage of squared bias showed marginal difference between models with WD function being relatively more biased than DIJ function. Correlation between actual and predicted test day milk yield indicate similar predictive accuracy between the functions.

TABLE III- ESTIMATES OF MEAN SQUARED PREDICTION ERROR (MSPE), PERCENTAGE OF SQUARED BIAS (PSB) AND CORRELATION FOR WD AND DIJ FUNCTIONS

		MSPE		PSB		Correlation	
	Parity	WD	DIJ	WD	DIJ	WD	DIJ
Guernsey	1	6.57	6.54	6.59	6.56	0.51	0.51
	2	7.31	7.30	6.373	6.37	0.60	0.60
	3	7.50	7.49	5.53	5.52	0.66	0.66
Jersey	1	25.31	CF	12.72	CF	0.27	CF
	2	25.33	25.32	11.89	11.88	0.46	0.47
	3	23.39	23.47	10.31	10.35	0.51	0.51
Sahiwal	1	3.99	3.95	19.76	19.55	0.50	0.51
	2	3.68	CF	16.89	CF	0.62	CF
	3	3.87	3.861	16.86	16.83	0.59	0.60

The WD and DIJ functions showed superior fit to the data. DIJ function owing to easy interpretation of its parameters would be an ideal function for describing lactation performance of dairy cattle. However due to its high data requirements restricts its use in smallholder systems. The WD function showed high adaptability to the various data characteristics hence the most appropriate function for describing lactation of dairy cattle in the smallholder systems where recording is not consistently practised.

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